

Precalculus algebra

Section 3.3: Rates of Change and Behavior of Graphs



What we will cover:

- ▶ Average rate of change
- ▶ Local maxima and local minima
- ▶ Absolute maximum and absolute minimum
- ▶ Locating and reporting where a function is increasing, decreasing and constant



Average rate of change

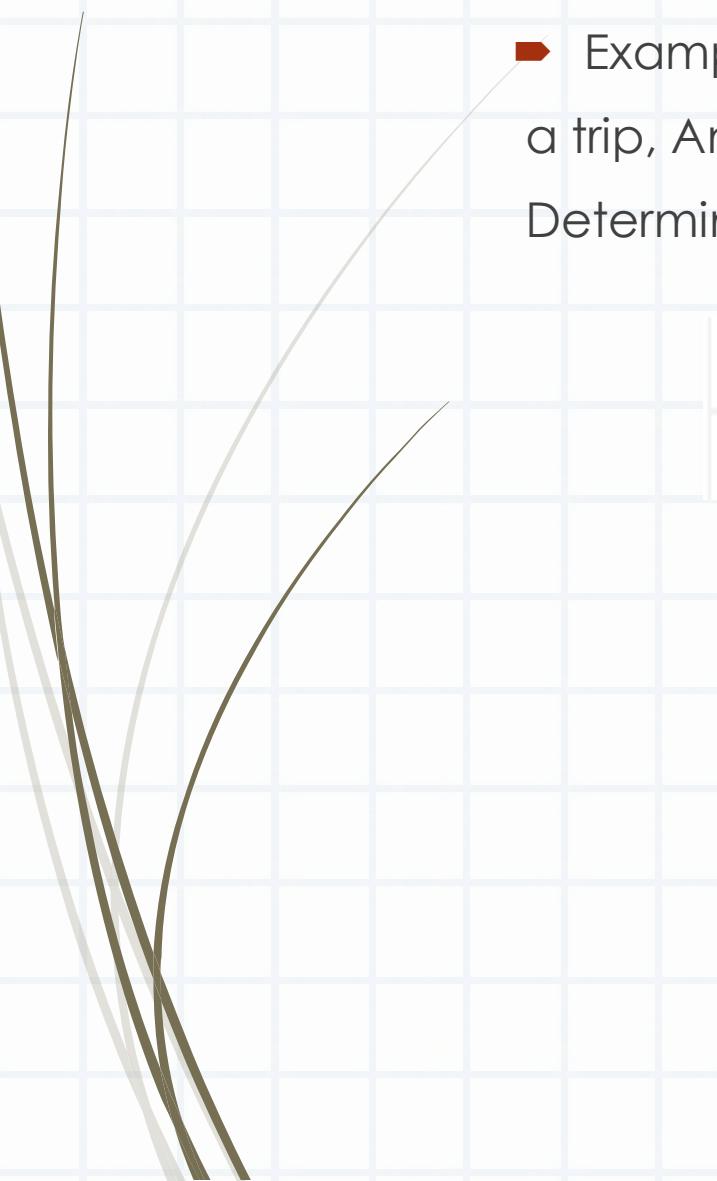
- ▶ Average rate of change tells how the output quantity changes with respect to the input quantity

- ▶ Average rate of change is identical to slope and has formula:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

- ▶ Example: If y axis represents miles travelled and x axis represents hours, then average rate of change tells distance travelled each hour.

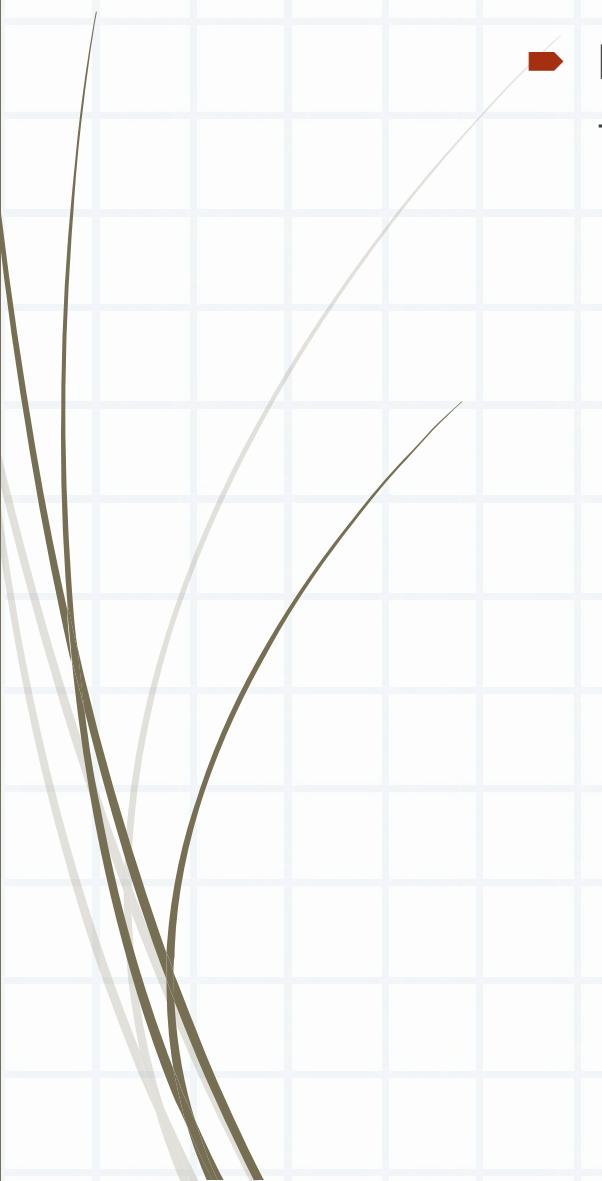
Average Rate of Change



- Example: After picking up a friend who lives 10 miles away and leaving on a trip, Anna records her distance from home over time (see below). Determine Anna's average speed over the first 6 hours.

t (hours)	0	1	2	3	4	5	6	7
$D(t)$ (miles)	10	55	90	153	214	240	292	300

Average Rate of Change

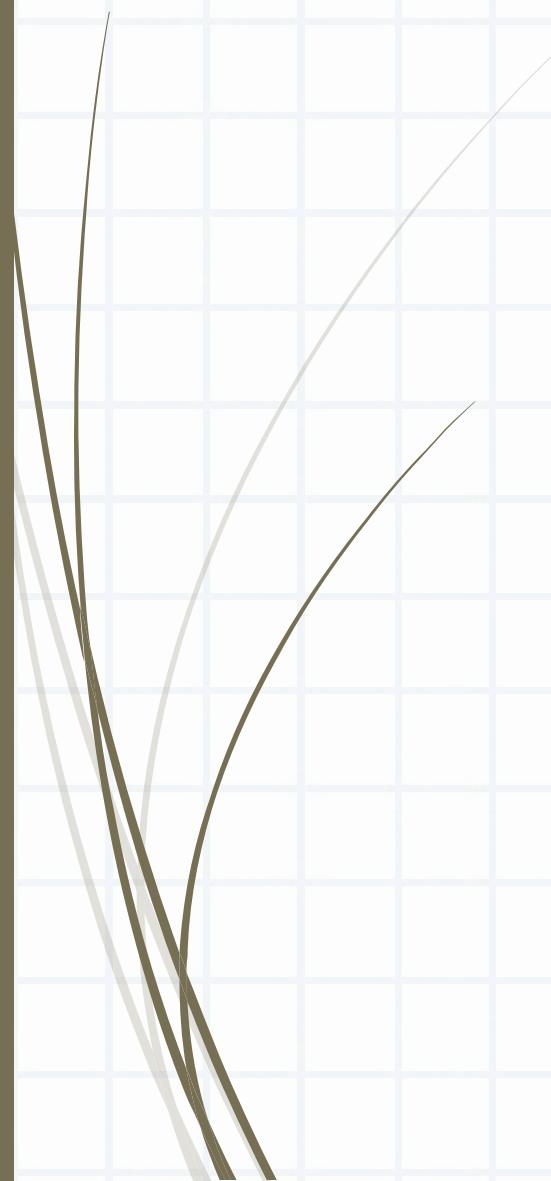


- ▶ Determine the average rate of change of the function $k(t) = 6t^2 + \frac{4}{t^3}$ on the interval $[-1,3]$



Average Rate of Change

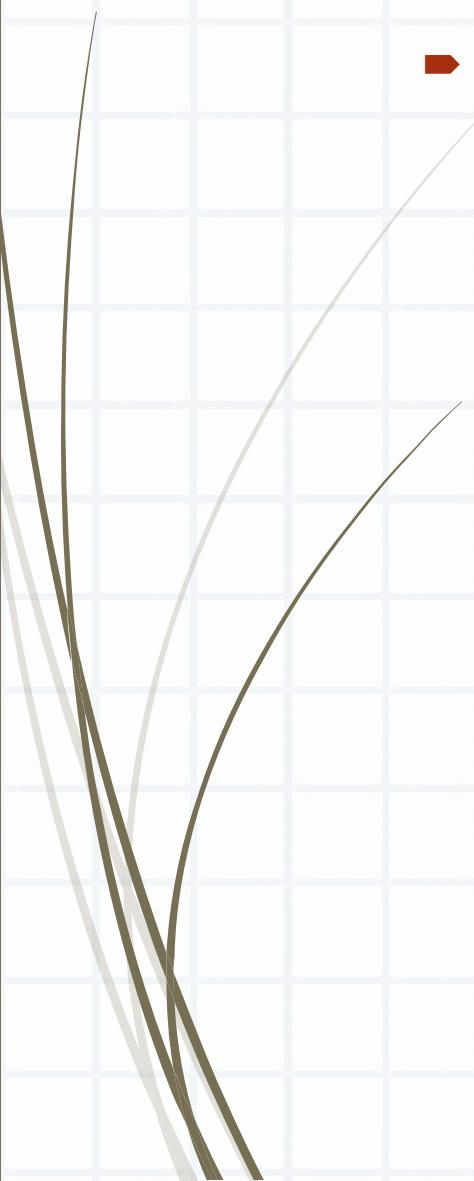
- ▶ Determine the average rate of change of the function $f(x) = 2x^2 + 1$ on the interval $[x, x + h]$



Determine where a function is Increasing

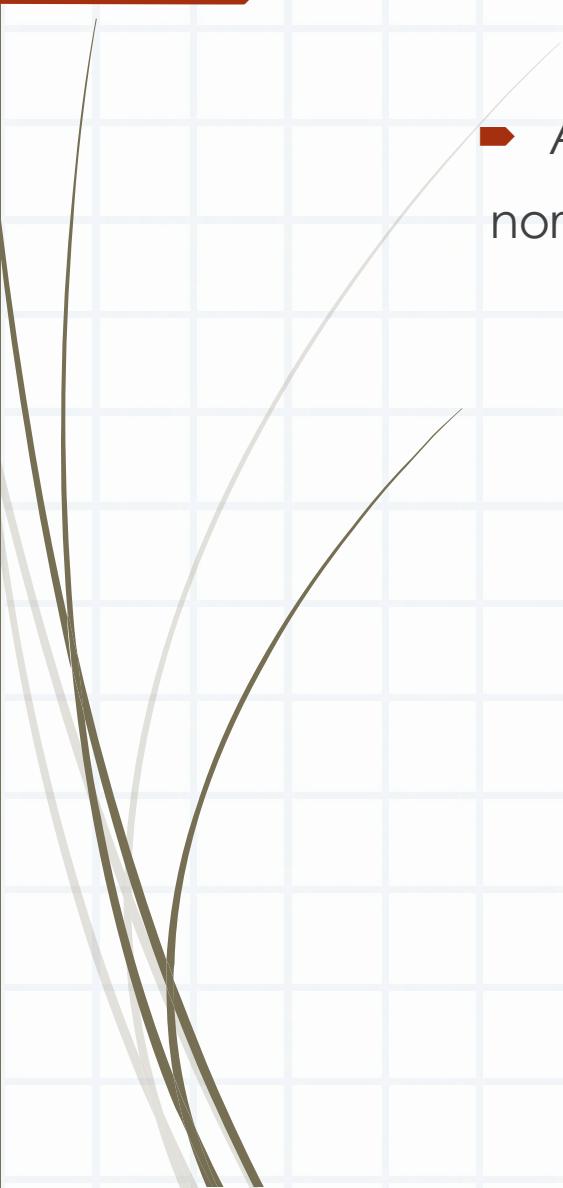
- ▶ A function is increasing on an interval if the output values increase as the input values increase within that interval
- ▶ A function that is increasing on an interval slants upward as we move from left to right
- ▶ Average rate of change (slope) remains positive on that interval

Determine where a function is decreasing



- A function is decreasing on an interval if the output values decrease as the input values increase within that interval
- A function that is decreasing on an interval slants downward as we move from left to right
- Average rate of change (slope) remains negative on that interval

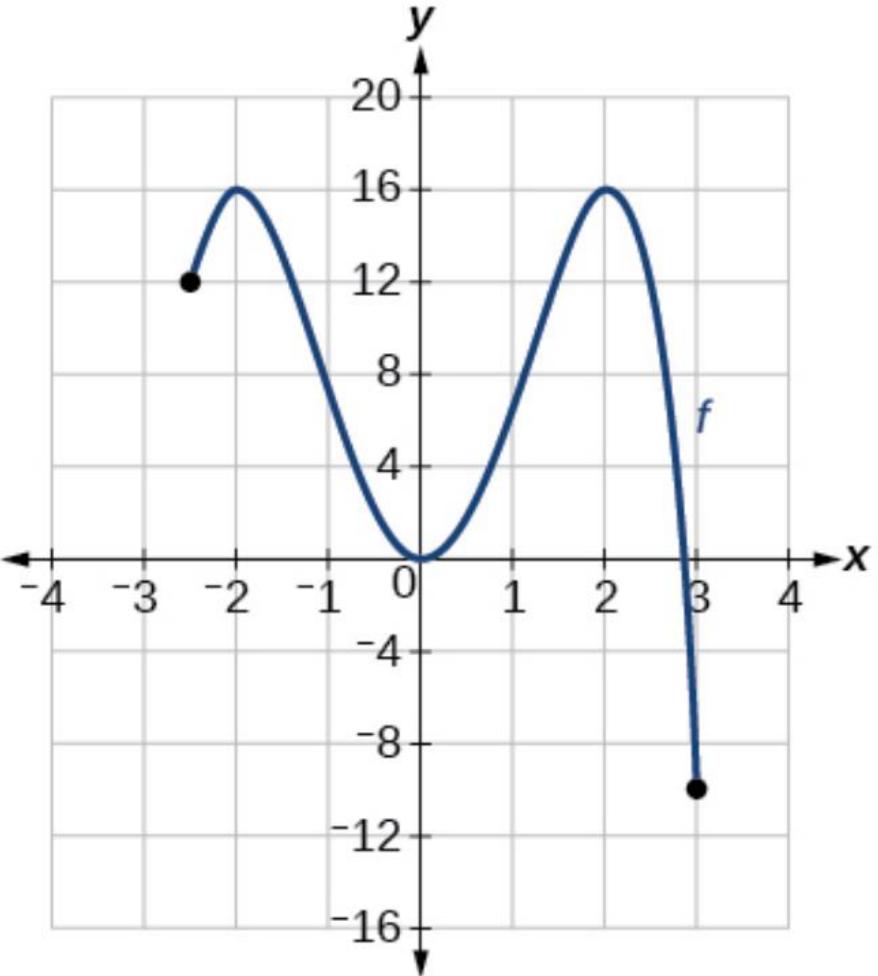
Determine where a function is constant



- ▶ A function is constant on an interval if the output values neither increase nor decrease as the input values increase within that interval
- ▶ A function that is a constant on an interval remains “flat” as we move from left to right
- ▶ Average rate of change (slope) remains zero on that interval

Question

- Estimate the intervals on which the function below is increasing or decreasing





Local Maxima (Local Extrema)

- A function f has a local maximum at $x = b$ if there exists an **open interval** containing b such that for any x in the open interval, $f(x) \leq f(b)$
- A potential local maximum is present where a function makes a transition from increasing to decreasing
- Local maximum **cannot** exist at the endpoint of function.



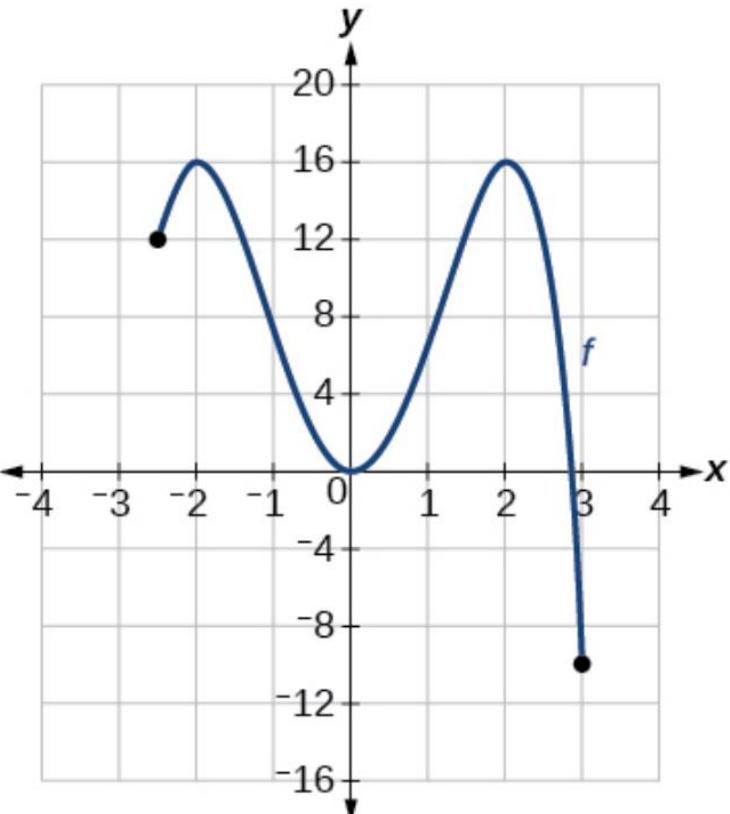
Local Minimum (Local Extrema)

- ▶ A function f has a local minimum at $x = b$ if there exists **an open interval** containing b such that for any x in the open interval, $f(b) \leq f(x)$
- ▶ A potential local minimum is present where a function makes a transition from decreasing to increasing.
- ▶ Local minimum **cannot** exist at the endpoint of function.

Question

► For the function below :

- State the domain of the function
- determine all local maxima and local minima



Absolute Extrema

- ▶ Absolute maximum
 - ▶ Largest value on the domain of function
- ▶ Formal definition:

The absolute maximum of f at $x = c$ is $f(c)$ where $f(c) \geq f(x)$ for all x in the domain of f .

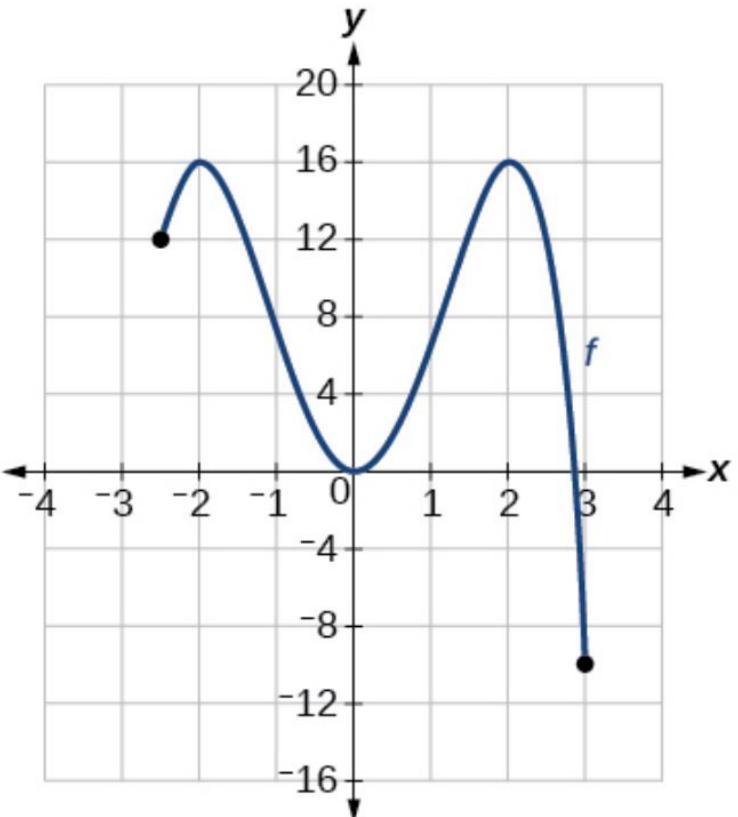
- ▶ Absolute minimum
 - ▶ Smallest value on the domain of function
- ▶ Formal definition:

The absolute minimum of f at $x = d$ is $f(d)$ where $f(d) \leq f(x)$ for all x in the domain of f .

- ▶ Note: It is possible for a function to have one absolute extrema located at different points on its domain.

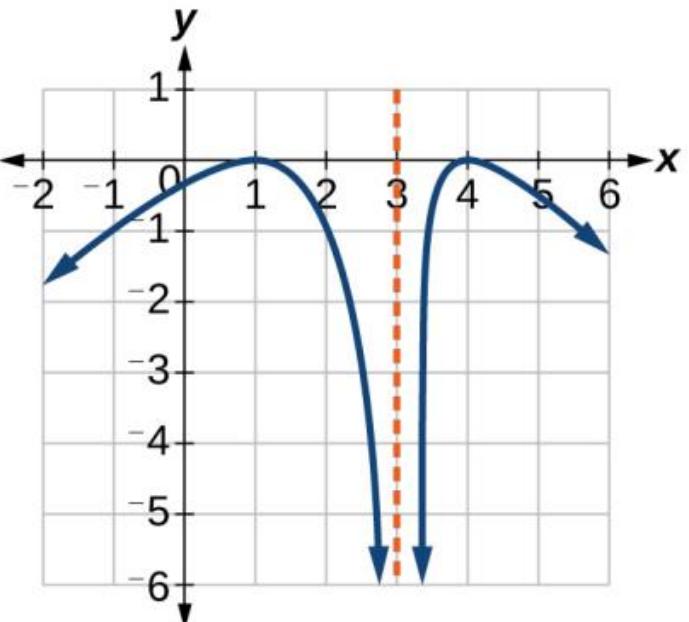
Question

- For the function below determine all absolute extrema



Question:

- For the function below determine:
 - the intervals on which the function is increasing or decreasing
 - the function's local extrema, if any
 - the function's absolute extrema, if any



Homework section 3.3 due:

- ▶ Review for test 1 (2/12):

Chapter 3 Practice Quiz (week 5) moodle, #1, 2, 3, 8, 11

Chapter 2 Practice Quiz (week 4) moodle, all problems

Good idea to look at homework problems from chapter 2 and 3.1-3.3

You will need a scientific calculator

cell phones calculators are not allowed.